1 Recursive

For each of the following functions, give the tightest asymptotic runtime bound.

(a) Hint: Draw out the recursive tree!

```
void f(int N) {
    if (N <= 1) {
        return;
    }
    f(N - 1);
    f(N - 1);
    f(N - 1);
}</pre>
```

(b) Assume that constantWork(N) runs in constant $\Theta(1)$ time.

```
void f(int N) {
    if (N <= 1) {
        return;
    }
    constantWork(N); // Runs in constant time
    f(N / 2);
    f(N / 2);
}</pre>
```

(c) What is the best and worst case runtime of alpha?

```
public void alpha(int N) {
    if (N % 4 == 0 || N <= 0) {
        return;
    }
    alpha(N - 4);
}</pre>
```

Best Case:

\bigcirc $\Theta(1)$	$\bigcap \ \Theta(\log N)$	$\bigcirc \ \Theta(N^2)$	$\bigcirc \ \Theta(N^3 \log N)$
$\bigcap \Theta(\log(\log N))$	$\bigcirc \ \Theta(N)$	$\bigcirc \ \Theta(N^2 \log N)$	$ \bigcirc \frac{\text{Worse than}}{\Theta(N^3 \log N)} $
$ \bigcirc \ \Theta \big((\log N)^2 \big)$	$ \bigcirc \ \Theta(N \log N)$	$\bigcirc \ \Theta(N^3)$	$ \bigcirc_{\text{(infinite loop)}}^{\text{Never terminates}} $

Worst Case:

Vorst Case:			
$\bigcirc \ \Theta(1)$	$ \bigcirc \ \Theta(\log N)$	\bigcap $\Theta(N^2)$	$\bigcirc \ \Theta(N^3 \log N)$
$\bigcirc \ \Theta(\log(\log N))$	$\bigcirc \ \Theta(N)$	$\bigcirc \ \Theta(N^2 \log N)$	$igcomes_{\Theta(N^3\log N)}^{ ext{Worse than}}$
$ \bigcirc \ \Theta \big((\log N)^2 \big)$	$ \bigcirc \ \Theta(N \log N)$	$\bigcirc \ \Theta(N^3)$	$O_{\text{(infinite loop)}}^{\text{Never terminates}}$

This page was intentionally left blank.

2 Comparisons Between Combinations

(a) Consider the four cases below, and decide whether the Comparable or Comparator interface is better suited for the job.

We are writing a custom **61BStudent** class, and want to define the default comparison behavior to be a comparison between their **Integer studentIDs**.

Comparator

O Comparable

We are writing a custom Dog class with many different attributes, including size, age, and timesBarked. We want to be able to sort 3 lists of Dogs by each of these attributes separately.

○ Comparable ○ Comparator

We have a list of **String**s and want to sort them by an **Integer** frequency value stored in an external **TreeMap** (sound familiar?)

○ Comparable ○ Comparator

We are writing a custom static sorting method that takes in a **List** and destructively sorts it. We want to be able to sort **anything** that has a predefined natural order.

O Comparable O Comparator

(b) Consider the City and CityPair classes below.

```
public class City {
  private String name;
  private double x;
 private double y;
 private int ranking;
  /* Constructor not shown...*/
public class CityPair {
  private City a;
  private City b;
  public CityPair(City a, City b) {
    this.a = a;
    this.b = b;
  }
  @Override
  public boolean equals(Object other) {
    // Two CityPairs are equal if they contain the same two cities.
    if (other instanceof CityPair p) {
      return ((this.a == p.a) && (this.b == p.b)) ||
      ((this.a == p.b) \&\& (this.b == p.a));
    }
    return false;
}
```

Implement the static method proximitySort, that takes in a Set of cities and returns a List of CityPair objects, sorted from closest to furthest for every possible pair of distinct cities. Assume proximitySort is in the same file as City and CityPair. Assume that CityPair has a valid and good hash function.

Additionally, implement the pairDistance helper method and DistanceComparator nested class. DistanceComparator compares CityPair objects such that CityPairs with a lower Euclidean distance between their respective City objects are considered "less than" those with a greater Euclidean distance.

Remember that the Euclidean distance formula is $\sqrt{\left(x_2-x_1\right)^2+\left(y_2-y_1\right)^2}$ for two points $(x_1,x_2), (y_1,y_2)$

The square root function Math.sqrt(double a) and the power function Math.pow(double base, int exponent) may come in handy.

List.sort takes in a Comparator, and returns a sorted list according to the ordering defined by the comparator.

```
public static List<CityPair> proximitySort(Set<City> cities) {
     ____<CityPair> allCombinations = _____
 if (_____) {
      allCombinations.add(_____);
  }
 }
        __<CityPair> orderedCombinations = new _____(_
 return orderedCombinations.sort(______)
}
public static double pairDistance(CityPair p) {
 // Returns the euclidean distance between the two cities represented by a CityPair
 return
}
public class DistanceComparator implements _____<CityPair> {
 @Override
 public int compare(CityPair p1, CityPair p2) {
   Double distance1 = pairDistance(p1);
   Double distance2 = pairDistance(p2);
   return _
 }
}
```

3 Filter

We want to make a FilteredList class that selects only certain elements of a List during iteration. To do so, we're going to use the Predicate interface defined below. Note that it has a method, test that takes in an argument and returns true if we want to keep this argument or false otherwise.

```
public interface Predicate<T> {
boolean test(T x);
}
```

For example, if L is any kind of object that implements List<String> (that is, the standard java.util.List), then FilteredList<String> FL = new FilteredList<>(L, filter); gives an iterable containing all items, x, in L for which filter.test(x) is true. Here, filter is of type Predicate.

Fill in the FilteredList class on the following page.

```
public class FilteredList<T> ______
    public FilteredList (List<T> L, Predicate<T> filter) {
    }
    @Override
    public Iterator<T> iterator() {
    }
    private \ class \ FilteredListIterator \ implements \ Iterator < T > \ \{
        public FilteredListIterator() {
        }
        @Override
        public boolean hasNext() {
        }
        @Override
        public T next() {
        }
```

}

}

4 A Tree Takes on Graphs

Your friend at Stanford has come to you for help on their homework! For each of the following statements, determine whether they are true or false; if false, provide counterexamples.

(a) "A graph with edges that all have the same weight will always have multiple MSTs."

(b) "No matter what heuristic you use, A* search will always find the correct shortest path."

(c) "If you add a constant factor to each edge in a graph, Dijkstra's algorithm will return the same shortest paths tree."