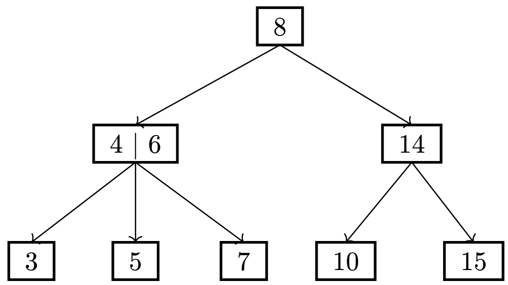
CS 61B — B-Trees, LLRBs, and Heaps

Fall 2025 — Discussion 07: October 13, 2025

1 2-3 Trees and LLRBs

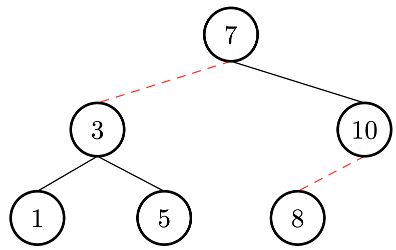
(a) Draw what the following 2-3 tree would look like after inserting 18, 38, 12, 13, and 20.



(b) Now, convert the resulting 2-3 tree to a left-leaning red-black tree.

(c) If a 2-3 tree has depth `H` (that is, the leaves are at distance `H` from the root), what is the maximum number of comparisons done in the corresponding red-black tree to find whether a certain key is present in the tree?

(d) Now, insert 9 into the LLRB Tree below. Describe where you would insert this node, and what balancing operations (**`rotateLeft`**, **`rotateRight`**, **`colorSwap`**) you'd take to balance the tree after insertion. Assume that in the given LLRB, dotted links between nodes are red and solid links between nodes are black.



2 Heap Mystery

We are given the following array representing a min-heap where each letter represents a **unique** number. Assume the root of the min-heap is at index one, i.e. `A` is the root. Our task is to figure out the numeric ordering of the letters. Therefore, there is **no** significance of the alphabetical ordering. i.e. just because B precedes C in the alphabet, we do not know if B is less than or greater than C.

Array: [-, A, B, C, D, E, F, G]

Four unknown operations are then executed on the min-heap. An operation is either a **`removeMin`** or an **`insert`**. The resulting state of the min-heap is shown below.

Array: [-, A, E, B, D, X, F, G]

(a) Determine the operations executed and their appropriate order. The first operation has already been filled in for you!

Hint: Which elements are gone? Which elements are newly added? Which elements are removed and then added back?

1. **`removeMin()`**
2. ______________________________
3. ______________________________
4. ______________________________

(b) Fill in the following comparisons with either >, <, or ? if unknown. We recommend considering which elements were compared to reach the final array.

1. X ________ D
2. X ________ C
3. B ________ C
4. G ________ X