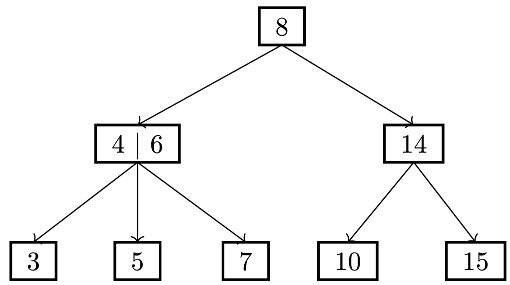
CS 61B | B-Trees, LLRBs, and Heaps

Fall 2025 | Discussion 07: October 13, 2025

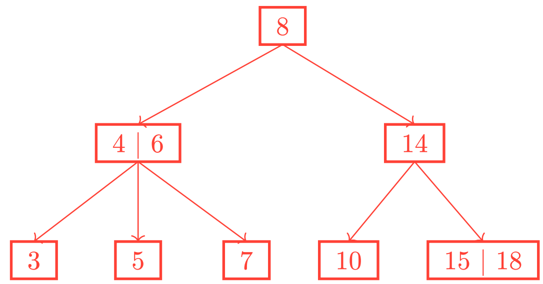
1 2-3 Trees and LLRBs

(a) Draw what the following 2-3 tree would look like after inserting 18, 38, 12, 13, and 20.

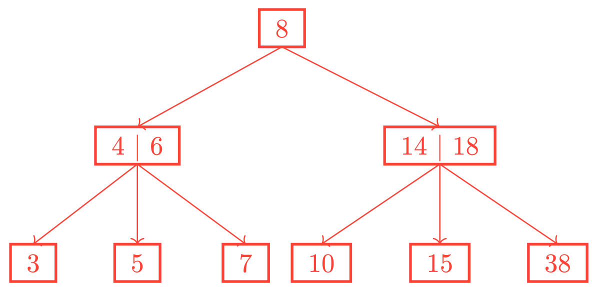


Solution:

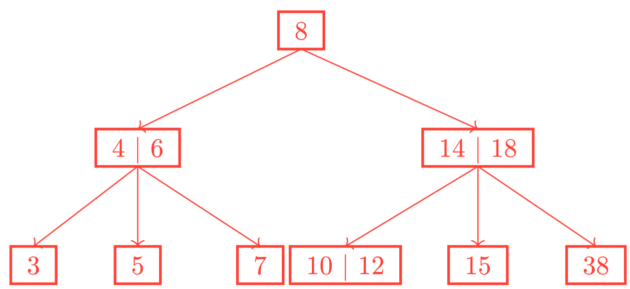
Adding 18:



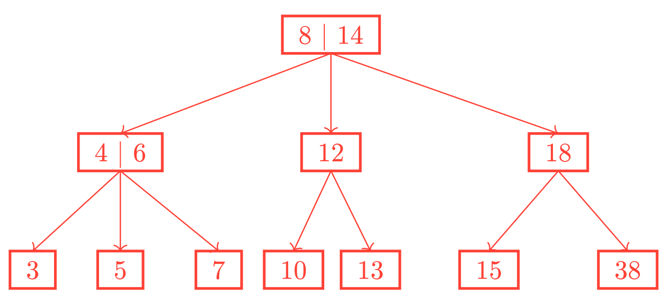
Adding 38:



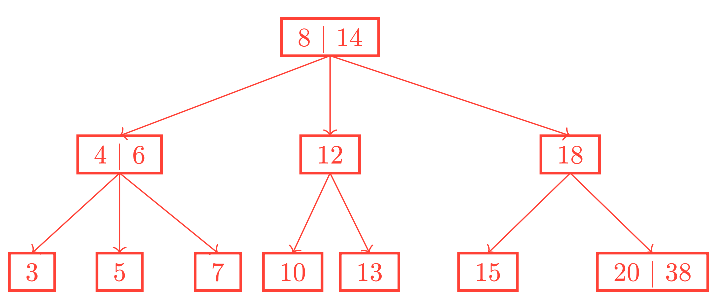
Adding 12:



Adding 13:

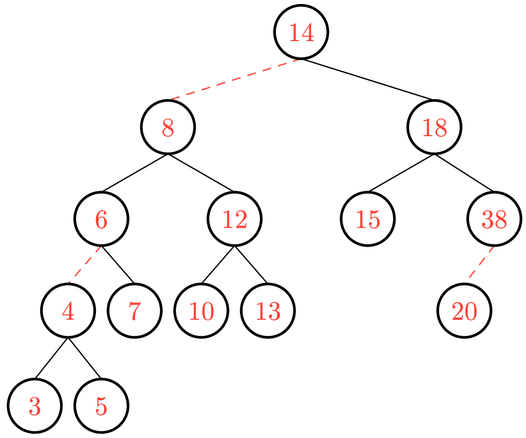


Adding 20:



(b) Now, convert the resulting 2-3 tree to a left-leaning red-black tree.

Solution:



(c) If a 2-3 tree has depth `H` (that is, the leaves are at distance `H` from the root), what is the maximum number of comparisons done in the corresponding red-black tree to find whether a certain key is present in the tree?

Solution:

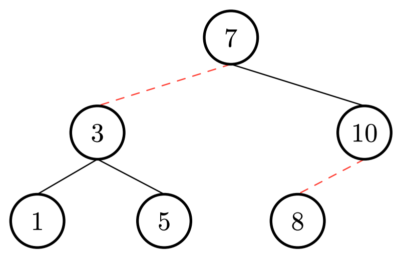
$2H + 2$ comparisons.

The maximum number of comparisons occur from a root to leaf path with the most nodes. Because the height of the tree is H, we know that there is a path down the leaf-leaning red-black tree that consists of at most H black links, for black links in the left-leaning red-black tree are the links that add to the height of the corresponding 2-3 tree. This means that there are $H + 1$ nodes on the path from the root to the leaf, since there is one less link than nodes.

In the worst case, in the 2-3 tree representation, this path can consist entirely of nodes with two items, meaning in the left-leaning red-black tree representation, each black link is followed by a red link. This doubles the amount of nodes on this path from the root to the leaf.

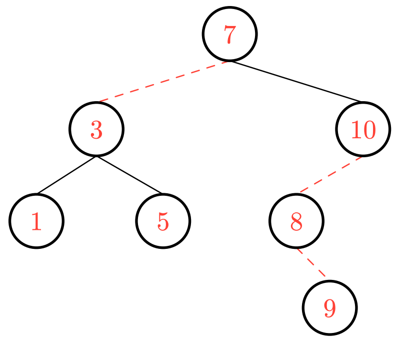
This example will represent our longest path, which is $2H + 2$ nodes long, meaning we make at most $2H + 2$ comparisons in the left-leaning red-black tree.

(d) Now, insert 9 into the LLRB Tree below. Describe where you would insert this node, and what balancing operations (**`rotateLeft`**, **`rotateRight`**, **`colorSwap`**) you'd take to balance the tree after insertion. Assume that in the given LLRB, dotted links between nodes are red and solid links between nodes are black.

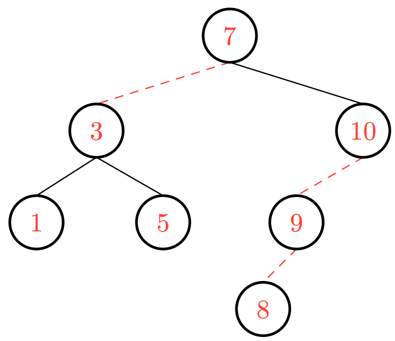


Solution:

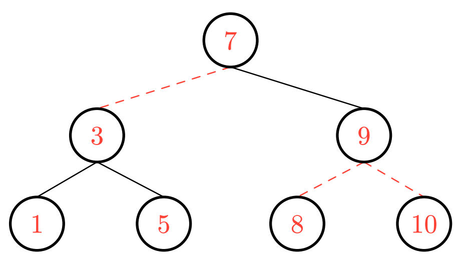
Remember that we always insert elements as leaf nodes with red links, so after initial insertion, the tree looks like:



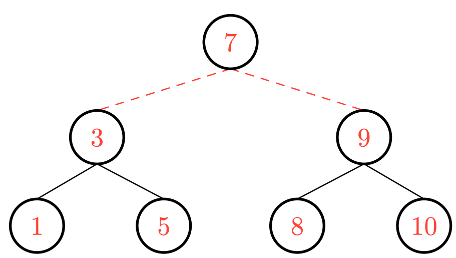
Now there is a right-leaning red link, so we will have to `rotateLeft(8)`:



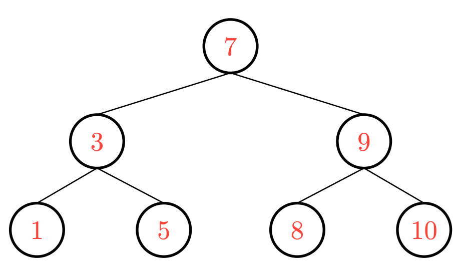
With the previous rotation, we now have two consecutive red left links, which makes our tree unbalanced. To fix, we `rotateRight(10)`:



Next, we have red links on both the left and the right, which can be fixed with a `colorFlip(9)`:



Finally, we still have red links on both the left and the right, which can be fixed with a `colorFlip(7)`:



2 Heap Mystery

We are given the following array representing a min-heap where each letter represents a **unique** number. Assume the root of the min-heap is at index one, i.e. **A** is the root. Our task is to figure out the numeric ordering of the letters. Therefore, there is **no** significance of the alphabetical ordering. i.e. just because B precedes C in the alphabet, we do not know if B is less than or greater than C.

Array: [-, A, B, C, D, E, F, G]

Four unknown operations are then executed on the min-heap. An operation is either a **`removeMin`** or an **`insert`**. The resulting state of the min-heap is shown below.

Array: [-, A, E, B, D, X, F, G]

(a) Determine the operations executed and their appropriate order. The first operation has already been filled in for you!

Hint: Which elements are gone? Which elements are newly added? Which elements are removed and then added back?

1. `removeMin()`
2. `insert(X)`
3. `removeMin()`
4. `insert(A)`

Explanation: We know immediately that A was removed. Then, after looking at the final state of the min-heap, we see that C was removed. Then, for A to remain in the min-heap, we see that A must have been inserted afterwards. And, after seeing a new value X in the min-heap, we see that X must have been inserted as well. We just need to determine the relative ordering of the **`insert(X)`** in between the operations **`removeMin()`** and **`insert(A)`**, and we see that the **`insert(X)`** must go before both.

(b) Fill in the following comparisons with either >, <, or ? if unknown. We recommend considering which elements were compared to reach the final array.

1. X ? D
2. X > C
3. B > C
4. G < X

Reasoning:
1. X is never compared to D
2. X must be greater than C since C is removed after X's insertion.
3. B must also be greater than C otherwise the second call to **`removeMin`** would have removed B
4. X must be greater than G so that it can be "promoted" to the top after the removal of C. It needs to be promoted to the top to land in its new position.